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Book Reviews

A Brief History of Numbers
By Leo Corry. Oxford University Press. 2015. 309+xiii pages

The history of mathematics is a subject of considerable scope. No one book can do justice to the entire sweep of time periods and topics, any more than a one-semester college-level survey course can cram in the entire subject. Whether author or teacher, anyone wanting to produce a successful but manageable account needs to be selective. In *A Brief History of Numbers*, Leo Corry puts together a satisfying and thorough history of mathematics book by limiting his attention to the development of numbers and the concept of number.

Corry's insightful book isn't written as a textbook, although I can see it being used successfully as an auxiliary resource for many university-level courses in the history of mathematics. It is too advanced for a general education audience, but it's entirely accessible to upper-division mathematics majors and therefore also appropriate for mathematics education students, at least those with more technical backgrounds. Indeed, Corry says that he had an undergraduate student of mathematics in mind as one typical reader when he was writing the book. He also considers the teacher of mathematics to be another typical reader, although he doesn't specify the level of this teacher. It would seem to be valuable background for those teaching anything from middle school to the undergraduate level.

Corry's first chapter begins with a brief overview of the aims of his book and the questions he hopes to address, both historical and philosophical. The remainder of the chapter is entirely technical and ahistorical: a review of the standard picture of numbers that has been accepted since the beginning of the 20th century or so. The typical reader will probably already be familiar with the Venn diagram of nested ellipses representing natural numbers, integers, rational, real and complex numbers, as well as the number line and the complex plane. He also pays attention to the distinction between algebraic and transcendental numbers and to the competing notions of cardinality and order. Although there is occasional mention here of historical figures, such as Dedekind or Cantor, this chapter is about setting the stage for the final four chapters of the book and establishing the technical vocabulary for the historical journey that will bring us there.

In a similar way, the second chapter begins with a technical discussion of positional number systems before presenting ancient systems of numeration. Decimal and non-decimal bases are considered as well as decimal fractions, explained using modern notation and terminology. Again, the goal is to appreciate how these ideas are understood today, before proceeding to contrast them with a short selection of early numeration schemes: Egyptian hieroglyphic, Babylonian cuneiform, and Greek cipher numerals.

The next two chapters concern the classical Greek mathematical tradition, with one chapter on numbers and magnitudes followed by another on construction problems and numerical problems. The fundamental notion of proportion is introduced here, first as understood by the Pythagoreans and subsequently as elaborated by Eudoxus. Corry carefully explains for the modern reader crucial and possibly unfamiliar distinctions, such as proportion vs. quotient, and comparison vs. measurement of magnitudes. His explanations are lucid and he avoids getting into excessive technical detail in the main body of his chapters. However, from the third chapter onward, with one exception, there is at least one appendix in which readers can get into the mathematical weeds if they so choose, but which can also be skipped over on a first reading without harming the conceptual flow of the book. In chapter 3, there are appendices on the incommensu-

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rability of $\sqrt{2}$, two propositions from Euclid's *Elements* using Eudoxus' theory of proportions (V.16 and V.18), and Euclid's proposition that circles are to one another as the squares of their diameters (XII.2). The following chapter covers Euclid's number theory, straightedge and compass constructions, and a detailed discussion of Diophantus' *Arithmetica*. As in the previous chapter, Corry takes great pain to contrast the Greek conception of the ideas involved in these works with the modern conception.

A Brief History of Numbers is almost exclusively a history of western mathematics. The one important exception is the chapter on numbers in the tradition of medieval Islam, which is the longest chapter in the book. Corry prefers the term "Islamicate mathematics" to refer to rich diversity of places, periods, and schools of thought in the regions in which Islam was culturally dominant during medieval times. Alongside Al-Khwārizmī, Abu Kāmil, and Al-Khayyām, readers may be surprised to find that he also considers the work of Gersonides (1288–1344), the Jewish scholar also known as Levy Ben Gerson, who was active in Provence. Although he lived outside of Islamic territory, he was part of a rich Hebrew mathematical tradition operating within Islamicate mathematics dating back at least to the 11th century. Corry closes out this chapter with a look at *Ma'aseh Hoshev*, an arithmetical text based on the number theory book of Euclid's *Elements*, but which included such modern innovations as a proof of the associative law for multiplication and the admission of the unit as a bona fide number. The example of Gersonides' proof of the formula for the sum of consecutive natural numbers gives Corry the opportunity to consider issues of the impact of the mathematical language in a particular historical context on its mathematical practice.

With the focus back on Europe, Corry next considers the gradual rise of algebra, and the corresponding birth of negative and imaginary numbers, in medieval Italy. European algebra comes of age at the beginning of the Scientific Revolution, at the same time that decimal fractions and logarithms are invented. With Descartes and Wallis, the primacy of geometry wanes and algebra takes center stage. This is also the period where western mathematics begins to abandon the classical theory of proportions in favor of the real numbers we use today. This three-chapter portion of Corry's book, spanning the twelfth to the seventeenth centuries, culminates with an examination of Newton's *Universal Arithmetick*.

The final four chapters cover the nineteenth century and the evolution of the real and complex number system into the scheme that the undergraduate reader will have learned in her upper division courses on algebra, analysis and foundations. The chapters cover complex numbers, real numbers and analysis, postulate systems for natural numbers, and set theory and the transfinite. This is the heart of Corry's book, what he has been working up to since the very first chapter. Readers who are already quite familiar with the history of mathematics could actually read these chapters as a stand-alone unit to great advantage.

A Brief History of Numbers is satisfying reading for anyone with an interest in the history of mathematics. For those with little background, it could serve as a useful introduction to the chronology of Western mathematics, as well as its major figures and schools of thought, even though great swaths of mathematics, including geometry and calculus, are largely absent. For Corry's notional undergraduate mathematics major, it would make an excellent basis for a self-directed reading course, although most instructors would probably want to supplement it with original source readings. It certainly deserves a place in an undergraduate mathematics library, where it can be used as an additional reference for a course in the history of mathematics.

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